

AD-A114 720

LEHIGH UNIV BETHLEHEM PA CENTER FOR THE APPLICATION--ETC F/6 20/11  
BIFURCATION IN A ELASTIC PLATE ON A RIGID SUBSTRATE.(U)

JAN 82 R S RIVLIN

N00014-76-C-0235

NL

UNCLASSIFIED

CAN-100-37

1 0

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

01 0000

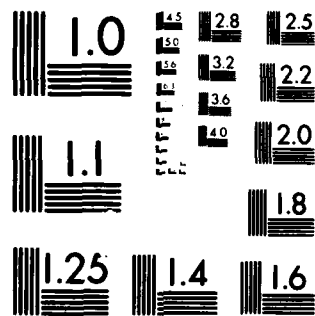
END

DATE

FILED

6 82

DTIC



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A



12  
**Lehigh  
University**

**BIFURCATION IN AN ELASTIC PLATE ON A RIGID SUBSTRATE**

by

**R.S. Rivlin**

TECHNICAL REPORT NO.

**CAM-100-37**

**January, 1982**

OFFICE OF NAVAL RESEARCH CONTRACT NO.

**N00014-76-C-0235**

DTIC FILE COPY

**DISTRIBUTION STATEMENT A**

Approved for public release;  
Distribution Unlimited

**Center for the  
Application of  
Mathematics  
(CAM)**

**82 05 21 036 :**

DTIC  
ELECTE  
MAY 21 1982

# Bifurcation in an Elastic Plate on a Rigid Substrate

by

R.S. Rivlin

Lehigh University, Bethlehem, Pa.

## Abstract

An infinite plate of neo-Hookean elastic material is bonded on one face to a rigid substrate. It is subjected to a uniform shear and dead-loaded with a uniform thrust. A periodic bifurcation solution is obtained when the thrust per unit area exceeds a critical value. The relation between the wave-length, thrust, amount of shear and plate thickness is obtained.



Accession For	
DTIS GRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/	
Availability Codes	
Avail and/or	
Dist	Special
A	

## 1. Introduction

In this paper we consider the critical loading conditions for which a bifurcation solution is obtained when an infinite plate of incompressible isotropic neo-Hookean elastic material, bonded on one face to a rigid substrate, is subjected to a uniform shear of amount  $K$  and simultaneously dead-loaded on the other face by a uniform normal thrust.

We suppose that an infinitesimal spatially periodic deformation is superposed. A secular equation is obtained for the determination of the wave-length of the superposed deformation. This secular equation yields a real value for the wave-length when some critical value of the normal thrust per unit area, which depends slightly on  $K$ , and is approximately equal to twice the shear modulus, is reached. For this critical value of the thrust, the wave-length of the infinitesimal superposed deformation is zero and, as the thrust is increased beyond this value,  $K$  remaining fixed, the wave-length increases.

Except at values of the thrust per unit area near the critical value, the wave-length is proportional to the square root of the thrust per unit area and is nearly independent of  $K$ . At values of the thrust near the critical value, the wave-length becomes extremely sensitive to the thrust. At all values of the thrust and of  $K$ , the wave-length is proportional to the thickness of the plate, as may be expected from dimensional considerations.

For specified values of the thickness and of the thrust per unit area, beyond the critical value of the latter, a periodic static bifurcation solution is obtained with uniquely

determined wave-length. However, if the plate were finite in the direction of the periodicity, the end conditions would enable us to determine the spectrum of values of the wave-length, and hence of the thrust per unit area, at which the bifurcation solution can occur.

## 2. Basic equations

We consider a flat plate of incompressible isotropic neo-Hookean elastic material of thickness  $h$  to be located with its major surfaces normal to the 2-axis of a rectangular Cartesian coordinate system  $x$ . The dimensions of the plate in the 1 and 3 directions are supposed large compared with  $h$ .

Let  $\xi$  be the vector position of a generic particle of the plate in its undeformed state and let  $\xi_\alpha (\alpha=1,2,3)$  be the components of  $\xi$  in the system  $x$ . We suppose that initially the plate is located with its major surfaces in the planes  $\xi_2 = 0$  and  $\xi_2 = h$  and that the face  $\xi_2 = 0$  is bonded to a rigid plate which remains fixed in space.

Suppose the elastic plate undergoes a deformation in which a particle initially at  $\xi$  moves to vector position  $x$  with components  $x_i (i=1,2,3)$  in the system  $x$ . Then, the strain-energy  $W$  per unit volume is given, in appropriately chosen units, by

$$W = \frac{1}{2} (x_{i,\alpha} x_{i,\alpha})^{-3}, \quad (2.1)$$

where  $_{,\alpha}$  is used to denote differentiation with respect to  $\xi_\alpha$ . The Piola-Kirchhoff stress  $\bar{\Pi}_{\alpha i}$ , referred to the system  $x$ , is given by

$$\bar{\Pi}_{\alpha i} = x_{i,\alpha} - \frac{1}{2} \bar{P} \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} x_{j,\beta} x_{k,\gamma}, \quad (2.2)$$

where  $\epsilon_{ijk}$  denotes the alternating symbol and  $\bar{P}$  is an arbitrary hydrostatic pressure. Since the material considered is

incompressible

$$\det |x_{i,\alpha}| = 1. \quad (2.3)$$

We now suppose that the deformation  $\xi \rightarrow x$  is the resultant of a finite deformation  $\xi \rightarrow X$  and an infinitesimal deformation  $X \rightarrow x$ , where

$$x = X + \epsilon u \quad (2.4)$$

and  $\epsilon$  is a small parameter. We assume that the force system associated with the deformation  $\xi \rightarrow x$  differs by terms of order  $\epsilon$  from that associated with the deformation  $\xi \rightarrow X$ . We accordingly write

$$\bar{\pi}_{\alpha i} = \bar{\pi}_{\alpha i} + \epsilon \pi_{\alpha i}, \quad \bar{p} = p + \epsilon p. \quad (2.5)$$

Then, from (2.2) we obtain, with (2.4) and (2.5),

$$\begin{aligned} \pi_{\alpha i} &= X_{i,\alpha} - \frac{1}{2} p \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} X_{j,\beta} X_{k,\gamma}, \\ \pi_{\alpha i} &= u_{i,\alpha} - \frac{1}{2} \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} \{ p (X_{k,\gamma} u_{j,\beta} + X_{j,\beta} u_{k,\gamma}) \\ &\quad + p X_{j,\beta} X_{k,\gamma} \}. \end{aligned} \quad (2.6)$$

Similarly from (2.3) we have

$$\det |X_{i,\alpha}| = 1, \quad \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} X_{i,\alpha} X_{j,\beta} u_{k,\gamma} = 0. \quad (2.7)$$



The Piola-Kirchhoff equations of equilibrium yield

$$\Pi_{\alpha i, \alpha} = 0 \quad \text{and} \quad \pi_{\alpha i, \alpha} = 0 . \quad (2.8)$$

### 3. The governing equations

If the deformation  $\xi \rightarrow X$  is a simple shear of amount  $K$ , for which the direction of shear is the 1-direction and the plane of shear is the 12-plane, then

$$X_1 = \xi_1 + K\xi_2, \quad X_2 = \xi_2, \quad X_3 = \xi_3. \quad (3.1)$$

We shall assume that the superposed infinitesimal deformation  $X \rightarrow x$  is a plane deformation in the 12-plane. We can then write

$$u_1 = u_1(\xi_1, \xi_2), \quad u_2 = u_2(\xi_1, \xi_2), \quad u_3 = 0. \quad (3.2)$$

With (3.1) equation (2.7)<sub>1</sub> is automatically satisfied and with (3.1) and (3.2) equation (2.7)<sub>2</sub> yields

$$u_{1,1} + u_{2,2} - Ku_{2,1} = 0. \quad (3.3)$$

Also, with (3.1), (3.2) and (3.3) equations (2.6) yield

$$\begin{aligned} \pi_{11} = \pi_{22} = \pi_{33} = 1 - P, \quad \pi_{12} = KP, \quad \pi_{21} = K, \\ \pi_{31} = \pi_{13} = \pi_{23} = \pi_{32} = 0, \end{aligned} \quad (3.4)$$

and

$$\begin{aligned} \pi_{11} = u_{1,1} - Pu_{2,2} - P, \quad \pi_{22} = u_{2,2} - Pu_{1,1} - P, \quad \pi_{3,3} = -P, \\ \pi_{12} = u_{2,1} + Pu_{1,2} + PK, \quad \pi_{21} = u_{1,2} + Pu_{2,1}, \quad \pi_{31} = \pi_{13} = \pi_{23} = \pi_{32} = 0. \end{aligned} \quad (3.5)$$

With (3.4), equation (2.8)<sub>1</sub> implies that  $P$  is constant. With (3.5), the incremental equation of equilibrium (2.8)<sub>2</sub> yields

$$u_{1,11} + u_{1,22} = p_{,1}, \quad u_{2,11} + u_{2,22} + Kp_{,1} = p_{,2}, \quad p_{,3} = 0. \quad (3.6)$$

The last of these equations implies that  $p = p(\xi_1, \xi_2)$ .

From (3.3) it follows that there exists a function  $\psi(\xi_1, \xi_2)$  in terms of which we may express  $u_1, u_2$  by the relations

$$u_1 = \psi_{,2} - K\psi_{,1}, \quad u_2 = -\psi_{,1}. \quad (3.7)$$

By substituting (3.7) in (3.6)<sub>1,2</sub> and eliminating  $p$ , we obtain

$$\nabla^2[(1+K^2)\psi_{,11} + \psi_{,22} - 2K\psi_{,12}] = 0, \quad (3.8)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial \xi_1^2} + \frac{\partial^2}{\partial \xi_2^2}. \quad (3.9)$$

We shall obtain solutions of (3.8) which are sinusoidal in the 1-direction. Accordingly, with the usual complex notation, we write

$$\psi = \phi(\xi_2)e^{ik\xi_1}, \quad (3.10)$$

where  $k$  is a constant. Introducing (3.10) into (3.8), we obtain

$$\phi'''' - 2ikK\phi''' - k^2(2+K^2)\phi'' + 2ik^3K\phi' + k^4(1+K^2)\phi = 0, \quad (3.11)$$

where the prime denotes differentiation with respect to  $\xi_2$ .

Equation (3.11) has the general solution

$$\phi = \sum_{A=1}^4 a_A e^{\alpha_A \xi_2}, \quad (3.12)$$

where the  $a$ 's are (complex) constants and the  $\alpha$ 's are given by

$$\alpha_1 = k, \quad \alpha_2 = -k, \quad \alpha_3 = k(1+iK), \quad \alpha_4 = k(-1+iK). \quad (3.13)$$

With (3.10) and (3.12), we obtain from (3.7)

$$\begin{aligned} u_1 &= e^{ik\xi_1} \sum_{A=1}^4 (\alpha_A - ikK) a_A e^{\alpha_A \xi_2}, \\ u_2 &= -ike^{ik\xi_1} \sum_{A=1}^4 a_A e^{\alpha_A \xi_2}. \end{aligned} \quad (3.14)$$

With (3.14), equations (3.6) yield

$$\begin{aligned} p_{,1} &= e^{ik\xi_1} \sum_{A=1}^4 (\alpha_A^2 - k^2) (\alpha_A - ikK) a_A e^{\alpha_A \xi_2}, \\ p_{,2} &= e^{ik\xi_1} \sum_{A=1}^4 (\alpha_A^2 - k^2) [-ik(1+K^2) + K\alpha_A] a_A e^{\alpha_A \xi_2}. \end{aligned} \quad (3.15)$$

These equations yield

$$p = -\frac{1}{K} e^{ik\xi_1} \sum_{A=1}^4 (\alpha_A^2 - k^2) (\alpha_A - ikK) a_A e^{\alpha_A \xi_2} + \text{constant}. \quad (3.16)$$

With (3.14) and (3.16), we obtain from (3.5)

$$\begin{aligned}
 \pi_{11} &= e^{ik\xi_1} \sum_{A=1}^4 \left\{ ik\alpha_A \left( P + \frac{\alpha_A^2}{k^2} \right) + K\alpha_A^2 \right\} a_A e^{\alpha_A \xi_2}, \\
 \pi_{22} &= e^{ik\xi_1} \sum_{A=1}^4 \left\{ -ik\alpha_A \left( 2 + P - \frac{\alpha_A^2}{k^2} \right) + K(\alpha_A^2 - k^2 - k^2 P) \right\} a_A e^{\alpha_A \xi_2}, \\
 \pi_{12} &= e^{ik\xi_1} \sum_{A=1}^4 \left\{ -ikK\alpha_A \left( P - 1 + \frac{\alpha_A^2}{k^2} \right) \right. \\
 &\quad \left. + k^2(1 + K^2) + \alpha_A^2(P - K^2) \right\} a_A e^{\alpha_A \xi_2}, \\
 \pi_{21} &= e^{ik\xi_1} \sum_{A=1}^4 \left\{ -ikK\alpha_A + (\alpha_A^2 + k^2 P) \right\} a_A e^{\alpha_A \xi_2}.
 \end{aligned} \tag{3.17}$$

#### 4. Solution of the secular equation

In this section we shall assume that the surface conditions on the surface  $\xi_2 = h$  are dead-loading conditions. Accordingly,

$$\pi_{22}|_{\xi_2=h} = \pi_{21}|_{\xi_2=h} = 0. \quad (4.1)$$

On the surface  $\xi_2 = 0$ , we have

$$u_1|_{\xi_2=0} = u_2|_{\xi_2=0} = 0. \quad (4.2)$$

Introducing (3.14) into (4.2), we obtain

$$\sum_{A=1}^4 (\alpha_A - ikK) a_A = 0, \quad \sum_{A=1}^4 a_A = 0. \quad (4.3)$$

Again introducing (3.17)<sub>2,4</sub> into (4.1), we obtain

$$\sum_{A=1}^4 B_A a_A = 0, \quad \sum_{A=1}^4 C_A a_A = 0, \quad (4.4)$$

where

$$\begin{aligned} B_A &= \left\{ -ik\alpha_A \left( 2+p - \frac{\alpha_A^2}{k^2} \right) + K(\alpha_A^2 - k^2 - k^2 p) \right\} e^{\alpha_A h}, \\ C_A &= \left\{ -ikK\alpha_A + \alpha_A^2 + k^2 p \right\} e^{\alpha_A h}. \end{aligned} \quad (4.5)$$

The necessary and sufficient condition for (4.3) and (4.4) to have a non-trivial solution for  $a_A$  is

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ B_1 & B_2 & B_3 & B_4 \\ C_1 & C_2 & C_3 & C_4 \end{vmatrix} = 0 . \quad (4.6)$$

With the expressions (4.5) for  $B_A$  and  $C_A$ , the secular equation (4.6) can be rewritten (see Appendix) as

$$\beta_1 Q^2 + 2\beta_2 Q + \beta_3 = 0 , \quad (4.7)$$

where

$$\begin{aligned} Q &= 1 + P , \\ \beta_1 &= (K^2 + 4) - (K^2 \cosh 2\mu + 4 \cos K\mu) , \\ \beta_2 &= 2K^2 (\cosh 2\mu - \cos K\mu) , \\ \beta_3 &= K^2 [(K^2 + 4) + (K^2 \cosh 2\mu + 4 \cos K\mu)] , \end{aligned} \quad (4.8)$$

with

$$\mu = kh . \quad (4.9)$$

From (4.7),  $Q$  is given by

$$Q = -\frac{1}{\beta_1} \left\{ \beta_2 \pm (\beta_2^2 - \beta_1 \beta_3)^{\frac{1}{2}} \right\} . \quad (4.10)$$

We note from (4.8) that

$$\begin{aligned}
\beta_2^2 - \beta_1 \beta_3 &= K^2(K^2+4)(K^2 \sinh^2 2\mu - 4 \sin^2 K\mu) , \\
\beta_1 &= -2(K^2 \sinh^2 \mu - 4 \sin^2 \frac{1}{2} K\mu) , \\
\beta_3 &= 2K^2(K^2 \cosh^2 \mu + 4 \cos^2 \frac{1}{2} K\mu) .
\end{aligned}
\tag{4.11}$$

Since  $K^2 \sinh^2 2\mu \geq 4 \sin^2 K\mu$  for all  $K$  and  $\mu$ , it follows from (4.10) and (4.11), that  $Q$  is real for all  $K$  and  $\mu$ . Also, it is evident from (4.11)<sub>2,3</sub> that  $\beta_1 \leq 0$  and  $\beta_3 \geq 0$ . It follows that the negative alternative in (4.10) leads to a negative value for  $Q$ . Since, from (3.4), the condition for the normal traction on  $\xi_2 = h$  to be a thrust is  $P > 1$ , i.e.  $Q > 2$ , it follows that the negative alternative in (4.10) corresponds to the normal traction on  $\xi_2 = h$  being tensile.

We note also that  $\beta_2 \geq 0$ . Accordingly, the necessary and sufficient condition that  $Q > 2$ , i.e. the normal traction is a thrust, is

$$(\beta_2^2 - \beta_1 \beta_3)^{\frac{1}{2}} > -2\beta_1 - \beta_2 . \tag{4.12}$$

From (4.8) it follows that

$$2\beta_1 + \beta_2 = 2(K^2+4)(1 - \cos K\mu) . \tag{4.13}$$

Accordingly, apart from the trivial case when  $\cos K\mu = 1$ , the inequality (4.12) is always satisfied by the positive alternative in (4.10). We conclude that this corresponds to the normal force on the surface  $\xi_2 = h$  being a thrust. In the remainder of this paper we consider only this situation.



In Fig.1 we plot from (4.10), with (4.8) and (4.11), the values of the thrust  $N = P-1=Q-2$  against  $\mu$  for  $K$  fixed. The curve in Fig.1 was drawn for  $K = 0.4$ . However, the dependence of  $N$  on  $K$ , for  $K$  in the range 0 to 0.4, is less than 2%. It was found from the numbers from which the figure was plotted that  $Q - 2 \approx 13.2/\mu^2$  for  $\mu < 1$ .

It is instructive to consider three limiting cases.

(a)  $\mu$  fixed,  $K \rightarrow 0$ .

From (4.8) it follows that

$$\begin{aligned}\beta_1 &= 2K^2(\mu^2 - \sinh^2\mu) + O(K^4), \\ \beta_2 &= 4K^2\sinh^2\mu + O(K^4), \\ \beta_3 &= 8K^2 + O(K^4).\end{aligned}\tag{4.14}$$

With these expressions we obtain from (4.10)

$$Q = \frac{2\{\sinh^2\mu + (\sinh^4\mu + \sinh^2\mu - \mu^2)^{1/2}\}}{\sinh^2\mu - \mu^2}.\tag{4.15}$$

In the limit  $\mu \rightarrow \infty$ ,  $Q = 4$ .

In the limit  $\mu \rightarrow 0$ ,  $Q = \frac{6}{\mu^2}(1 + \frac{2}{\sqrt{3}})$ .

(b)  $K$  fixed,  $\mu \rightarrow \infty$ .

From (4.8), it follows that, as  $\mu \rightarrow \infty$ ,

$$\begin{aligned}\beta_1 &= -\frac{1}{2}K^2e^{2\mu}, \\ \beta_2 &= K^2e^{2\mu}, \\ \beta_3 &= \frac{1}{2}K^4e^{2\mu}.\end{aligned}\tag{4.16}$$

Then, from (4.10), we obtain

$$Q = 2 + (K^2+4)^{\frac{1}{2}} . \quad (4.17)$$

As  $K$  increases from 0 to 1 ,  $Q|_{\mu=\infty}$  increases from 4 to 4.236.

(c)  $K$  fixed,  $\mu \rightarrow 0$  .

From (4.8), we obtain, as  $\mu \rightarrow 0$

$$\begin{aligned} \beta_1 &= -\frac{1}{6}\mu^4 K^2(K^2+4) + O(\mu^6) , \\ \beta_2 &= \mu^2 K^2(K^2+4) + O(\mu^4) , \\ \beta_3 &= 2K^2(K^2+4) + O(\mu^2) . \end{aligned} \quad (4.18)$$

Then, from (4.10), we obtain

$$Q = \frac{6}{\mu^2} \left(1 + \frac{2}{\sqrt{3}}\right) \approx 12.93/\mu^2 , \quad (4.19)$$

in agreement with the result obtained in case (a).

### 5. Appendix

With the expressions (3.13) for  $\alpha_A$ , equation (4.6) can be written as

$$\begin{aligned} & (B_1 C_2 - B_2 C_1) + (B_2 C_3 - B_3 C_2) + (B_3 C_4 - B_4 C_3) + (B_4 C_1 - B_1 C_4) \\ & + \frac{1}{2} i K \{ (B_1 - B_2)(C_3 - C_4) - (C_1 - C_2)(B_3 - B_4) \} = 0 . \end{aligned} \quad (5.1)$$

Also, with (3.13) and the notation  $Q = 1 + P$  and  $\mu = kh$ , we obtain from (4.5)

$$\begin{aligned} B_1 &= k^2 [K(1-Q) - iQ] e^\mu , \\ B_2 &= k^2 [K(1-Q) + iQ] e^{-\mu} , \\ B_3 &= -k^2 [iQ + K(1+iK)] e^{(1+iK)\mu} , \\ B_4 &= k^2 [iQ - K(1-iK)] e^{-(1-iK)\mu} , \\ C_1 &= k^2 (Q - iK) e^\mu , \\ C_2 &= k^2 (Q + iK) e^{-\mu} , \\ C_3 &= k^2 (Q + iK) e^{(1+iK)\mu} , \\ C_4 &= k^2 (Q - iK) e^{-(1-iK)\mu} . \end{aligned} \quad (5.2)$$

From (5.2) we obtain

$$\begin{aligned} B_3 C_4 - B_4 C_3 &= (B_1 C_2 - B_2 C_1) e^{2ik\mu} = -2ik^4 \{ Q + (Q-1)K^2 \} e^{2ik\mu} , \\ B_2 C_3 - B_3 C_2 &= k^4 (Q + iK) \{ 2iQ + K(1-Q) + K(1+iK) \} e^{iK\mu} , \\ B_4 C_1 - B_1 C_4 &= k^4 (Q - iK) \{ 2iQ - K(1-Q) - K(1-iK) \} e^{iK\mu} . \end{aligned} \quad (5.3)$$

Whence,

$$(B_2C_3 - B_3C_2) + (B_4C_1 - B_1C_4) = 4ik^4(Q^2 + K^2)e^{iK\mu} . \quad (5.4)$$

We also obtain from (5.2)

$$\begin{aligned} & (B_1 - B_2)(C_3 - C_4) - (C_1 - C_2)(B_3 - B_4) \\ & = k^4 \{ [QK(4-Q) + K^3](e^{2\mu} + e^{-2\mu}) + 2K(Q^2 + K^2) \} e^{iK\mu} . \end{aligned} \quad (5.5)$$

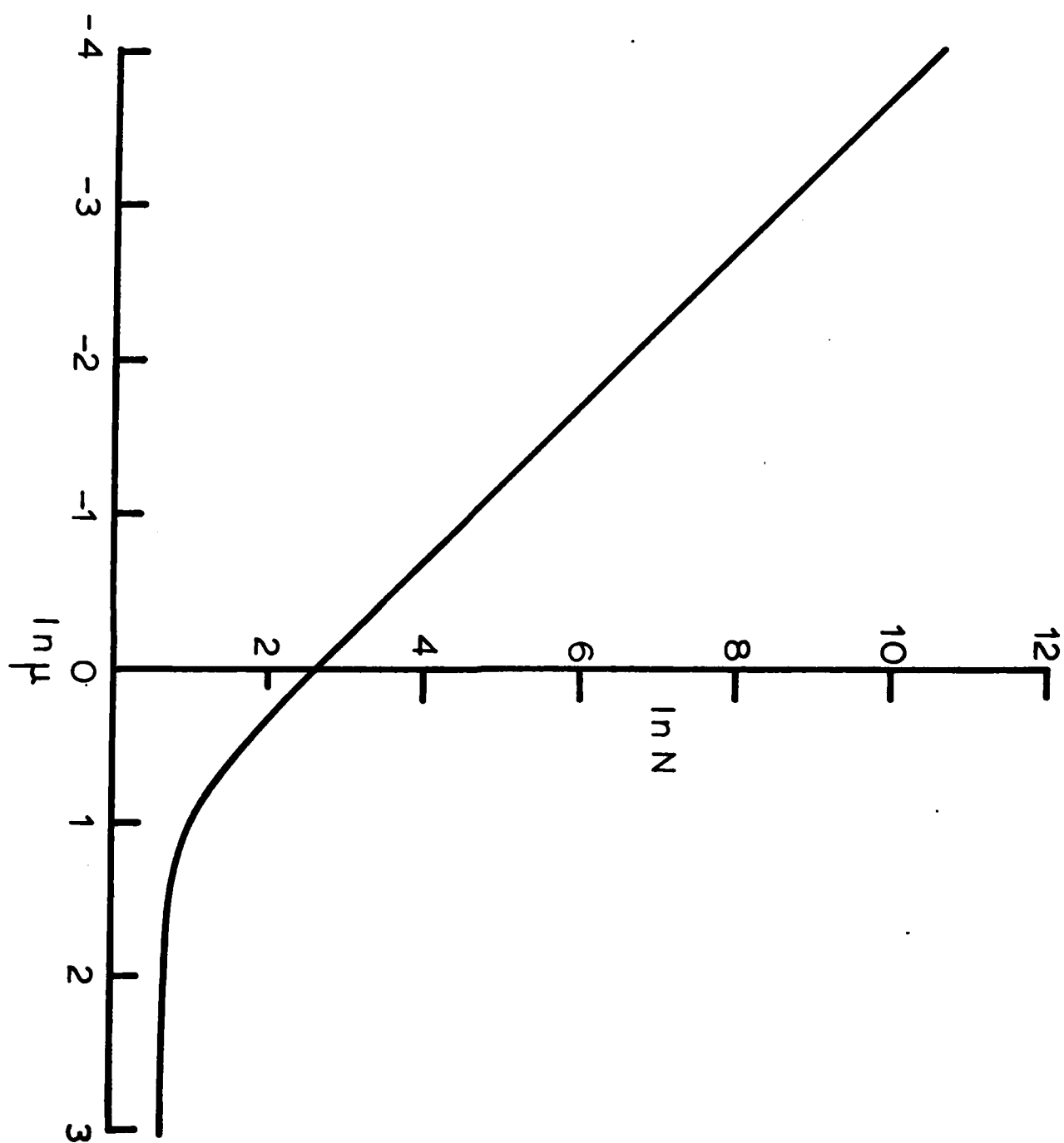
With (5.3), (5.4) and (5.5), the secular equation (5.1) may be rewritten as

$$\begin{aligned} & -2\{Q^2 + (Q-1)K^2\}(1 + e^{2iK\mu}) + 4(Q^2 + K^2)e^{iK\mu} \\ & + \frac{1}{2}K^2 \{ [Q(4-Q) + K^2](e^{2\mu} + e^{-2\mu}) + 2(Q^2 + K^2) \} e^{iK\mu} = 0 . \end{aligned} \quad (5.6)$$

From this the relation (4.7) with (4.8) is easily obtained.

Acknowledgement

This paper was written with the partial support of the Office of Naval Research under Contract No.00014-76-C-0235 with Lehigh University.



Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CAM-100-37	2. GOVT ACCESSION NO. AD A114720	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Bifurcation in an Elastic Plate on a Rigid Substrate		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R.S. Rivlin		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0235
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lehigh University Center for the Application of Mathematics Bethlehem, Pa. 18015		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA.		12. REPORT DATE January 1982
		13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same as above		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Distribution of this document is unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Finite elasticity, layered media, buckling		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An infinite plate of neo-Hookean elastic material is bonded on one face to a rigid substrate. It is subjected to a uniform shear and dead-loaded with a uniform thrust. A periodic bifurcation solution is obtained when the thrust per unit area exceeds a critical value. The relation between the wave-length, thrust, amount of shear and plate thickness is obtained.		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 68 IS OBSOLETE  
S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DATE  
FILME  
— 8